

Trigonometry: The Law of Sines

The LAW OF SINES is a powerful triangle tool which is used to find missing sides or angles of ANY triangle. By matching up angles with their **opposite** sides, the equation is:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

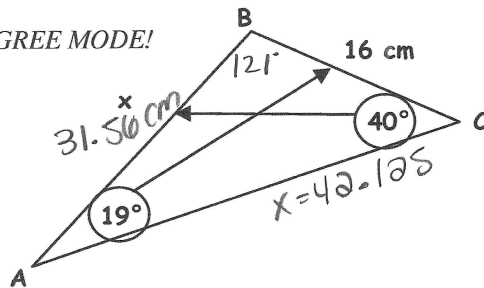
Example: Find the missing side x:

$$\frac{\sin 19^\circ}{16} = \frac{\sin 40^\circ}{x} \text{ DEGREE MODE!}$$

$$\frac{.326}{16} = \frac{.643}{x}$$

$$.326x = 10.288$$

$$x = 31.56 \text{ cm}$$



How about finding the other unknowns?

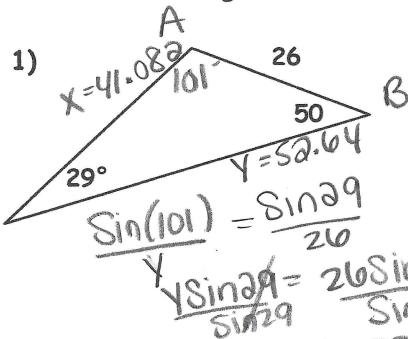
$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 121}{16} = \frac{\sin 19}{x}$$

$$\frac{16 \sin 121}{\sin 19} = \frac{16 \sin 19}{\sin 19}$$

$$x = 42.125$$

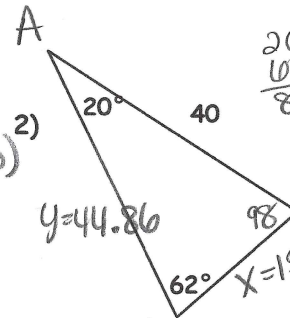
Solve each triangle:



$$\frac{\sin(50)}{x} = \frac{\sin 29}{26}$$

$$x \sin 29 = 26 \sin(50)$$

$$x = 41.082$$

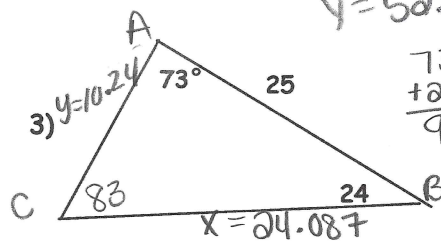


$$\frac{20}{\sin 20} = \frac{180}{\sin 98}$$

$$\frac{\sin 62}{40} = \frac{\sin 20}{x}$$

$$x \sin 62 = 40 \sin 20$$

$$x = 15.49$$



$$\frac{\sin 73}{x} = \frac{\sin 83}{25}$$

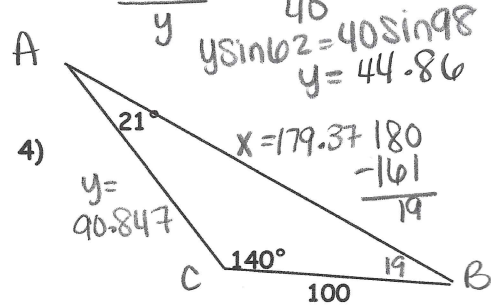
$$x \sin 83 = 25 \sin 73$$

$$x = 24.087$$

$$\frac{\sin 83}{25} = \frac{\sin 24}{y}$$

$$y \sin 83 = 25 \sin 24$$

$$y = 10.2448$$



$$\frac{\sin 21}{100} = \frac{\sin 140}{x}$$

$$x \sin 21 = 100 \sin 140$$

$$x = 179.37$$

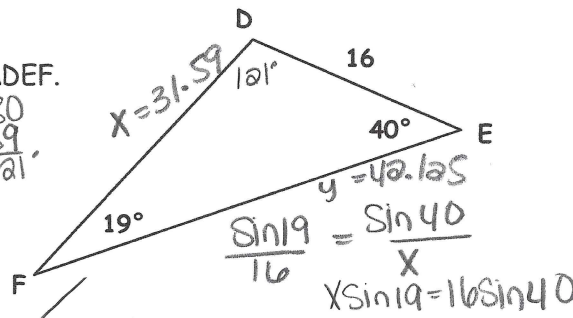
5) Find the perimeter of $\triangle DEF$.

$$\frac{\sin 19}{16} = \frac{\sin 121}{y}$$

$$y \sin 19 = 16 \sin 121$$

$$y = 42.125$$

$$\frac{180}{\sin 121} = \frac{180}{\sin 121}$$



$$\frac{\sin 19}{100} = \frac{\sin 19}{y}$$

$$y \sin 19 = 100 \sin 19$$

$$y = 90.847$$

This worksheet was adapted from <http://www.bgsd.k12.wa.us/riv/homework/Geometry/LawOfSines.doc>

Perimeter: $31.59 + 16 + 42.125 = 89.715$ $x = 31.590$

The LAW OF SINES can also be used to find missing angles.

Example: Find the missing angle x :

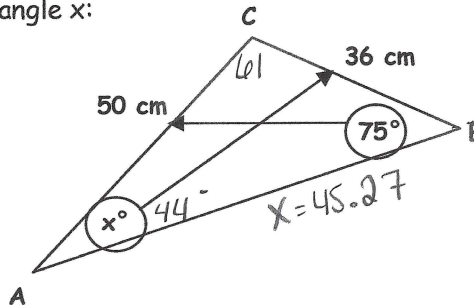
$$\frac{\sin x^\circ}{36} = \frac{\sin 75^\circ}{50}$$

$$\frac{\sin x^\circ}{36} = .966$$

$$50(\sin x^\circ) = 34.776$$

$$\sin x^\circ = .69532$$

$$x = 44^\circ \text{ (using inverse sine on your calculator)}$$



What about the other unknowns?

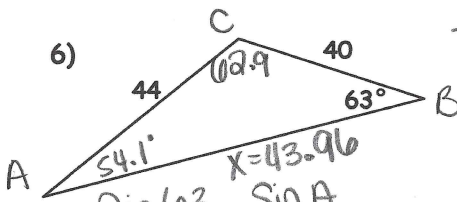
$$\begin{array}{r} 75 \\ +44 \\ \hline 119 \end{array} \quad \begin{array}{r} 180 \\ -119 \\ \hline 61 \end{array}$$

$$\frac{\sin 75}{50} = \frac{\sin 61}{x}$$

$$\frac{x \sin 75}{50 \sin 75} = \frac{50 \sin 61}{50 \sin 75}$$

$$x = 45.27$$

Solve each triangle:



$$\frac{\sin 63}{44} = \frac{\sin A}{40}$$

$$40 \sin 63 = 44 \sin A$$

$$.81 = \sin A$$

$$A = \sin^{-1}(.81)$$

$$A = 54.0959$$

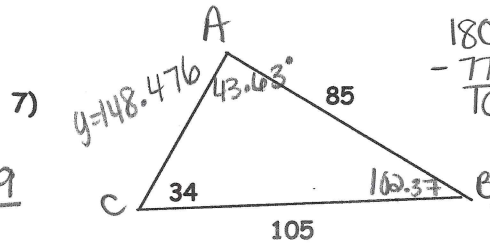
$$54.1 + 63 = 117.1$$

$$\begin{array}{r} 180 \\ -117.1 \\ \hline 62.9 \end{array}$$

$$\frac{\sin 63}{44} = \frac{\sin 62.9}{c}$$

$$c \sin 63 = 44 \sin 62.9$$

$$c = 43.96$$



$$\begin{array}{r} 180 \\ -77.63 \\ \hline 102.37 \end{array}$$

$$\frac{\sin 34}{85} = \frac{\sin A}{105}$$

$$105 \sin 34 = 85 \sin A$$

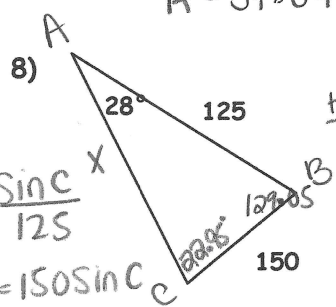
$$\sin A = .69$$

$$A = \sin^{-1}(.69) = 43.63^\circ$$

$$\frac{\sin 102.37}{y} = \frac{\sin 34}{85}$$

$$y \sin 34 = 85 \sin 102.37$$

$$y = 148.476$$



$$\frac{\sin 28}{150} = \frac{\sin C}{125}$$

$$125 \sin 28 = 150 \sin C$$

$$\sin C = .39$$

$$C = \sin^{-1}(.39)$$

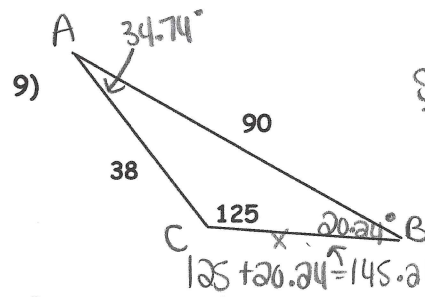
$$C = 22.95^\circ$$

$$\begin{array}{r} 28 \\ +22.95 \\ \hline 50.95 \end{array} \quad \begin{array}{r} 180 \\ -50.95 \\ \hline 129.05 \end{array}$$

$$\frac{\sin 129.05}{x} = \frac{\sin 28}{150}$$

$$150 \sin 129.05 = \frac{x \sin 28}{\sin 28}$$

$$x = 248.129$$



$$\begin{array}{r} 180 \\ -145.24 \\ \hline 34.74 \end{array}$$

$$\frac{\sin 125}{90} = \frac{\sin B}{38}$$

$$38 \sin 125 = 90 \sin B$$

$$\sin B = .346$$

$$B = \sin^{-1}(.346)$$

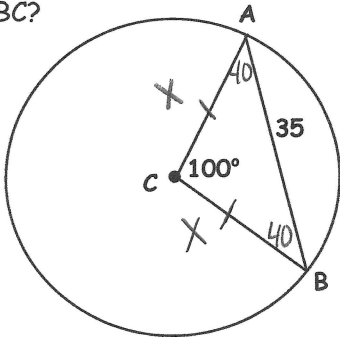
$$B = 20.24$$

$$\frac{\sin 125}{90} = \frac{\sin 34.74}{x}$$

$$x \sin 125 = 90 \sin 34.74$$

$$x = 62.61$$

10) Find the area of circle C by using the Law of Sines to find the radius. Hint: What kind of triangle is ABC?



$$\begin{array}{r} 180 \\ -100 \\ \hline 80 \div 2 = 40 \end{array}$$

$$\frac{\sin 100}{35} = \frac{\sin 40}{x}$$

$$x \sin 100 = 35 \sin 40$$

$$x = \frac{35 \sin 40}{\sin 100}$$

$$x = 22.845$$

$$A = \pi r^2$$

$$A = \pi (22.845)^2 = \pi \cdot 521.89 \approx 1639.56 \text{ units}^2$$