

Name: _____

6.4 Trigonometric Functions

Directions: Show all work. Justify all your answers algebraically; single answers will not receive credit. Draw diagrams and represent the angle measures on the diagrams when possible.

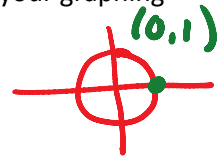
For questions 1-2, you may use your graphing calculator. Explain what you did on your graphing calculator, and why that makes sense in the context of the problem.

1. For what real numbers in $[0, 2\pi]$ does $\sin t = 1$?

- calc needs to be in radians.

when $y = 1$, $t = \frac{\pi}{2}$

$y_1 = \sin t$
 $y_2 = 1$



a. What are all the real numbers for which $\sin t = 1$?

The period of $\sin t$ is 2π

$\frac{\pi}{2} \pm 2\pi k$ such that k is an integer.

2. Find the domain and range of the cotangent function. Explain your reasoning.

The domain of tangent is all real #'s.

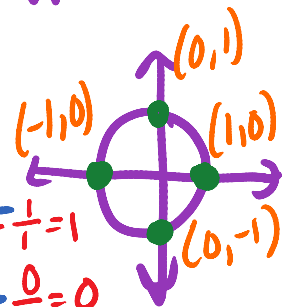
Range: \mathbb{R} so that $\theta \neq$ a multiple of π
(asymptote)

Cotangent: IS the inverse of tangent, so the domain & range are flipped

D: \mathbb{R} so that $\theta \neq$ a multiple of π
R: \mathbb{R}

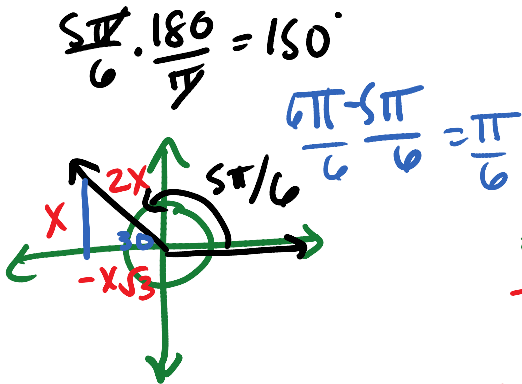
3. Find the exact values of the cosecant, secant, and cotangent functions when $t = \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$.

$\csc\left(\frac{\pi}{2}\right) = \frac{1}{1} = 1$ $\sec\left(\frac{\pi}{2}\right) = \frac{1}{0} = \text{und.}$ $\cot\left(\frac{\pi}{2}\right) = \frac{0}{1} = 0$
 $\csc\left(\frac{3\pi}{2}\right) = \frac{1}{-1} = -1$ $\sec\left(\frac{3\pi}{2}\right) = \frac{1}{0} = \text{und.}$ $\cot\left(\frac{3\pi}{2}\right) = \frac{0}{-1} = 0$
 $\csc(2\pi) = \frac{1}{0} = \text{und.}$ $\sec(2\pi) = \frac{1}{1} = 1$ $\cot(2\pi) = \frac{0}{0} = \text{und.}$



4. Find the exact value of the cosine, sine, and tangent of:

a.) $\frac{5\pi}{6}$

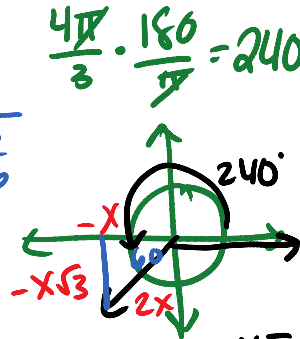


$\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{x}{2x} = \frac{1}{2}$

$\cos\left(\frac{5\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = \frac{-x\sqrt{3}}{2x} = -\frac{\sqrt{3}}{2}$

$\tan\left(\frac{5\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = \frac{-x}{x\sqrt{3}} = -\frac{\sqrt{3}}{3}$

b.) $\frac{4\pi}{3}$



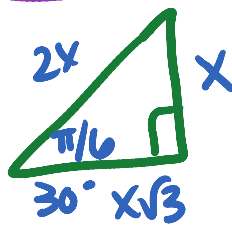
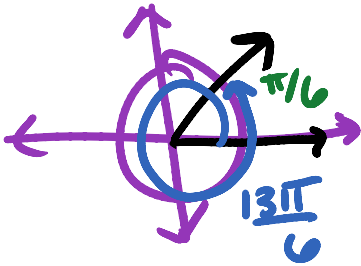
$\frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

$\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{x\sqrt{3}}{2x} = -\frac{\sqrt{3}}{2}$

$\cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{x}{2x} = -\frac{1}{2}$

c.) $\frac{13\pi}{6} \cdot \frac{180}{\pi} = 390^\circ$
Coterminal

$\frac{13\pi}{6} - \frac{12\pi}{6} = \frac{\pi}{6}$



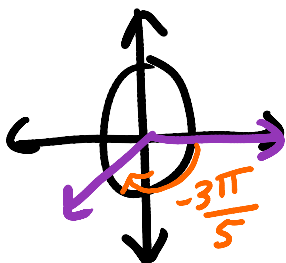
$\sin\left(\frac{\pi}{6}\right) = \frac{x}{2x} = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}\right) = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$

$\tan\left(\frac{\pi}{6}\right) = \frac{x}{x\sqrt{3}} = \frac{\sqrt{3}}{3}$

5. Sketch the angle, and give its reference angle in radians:

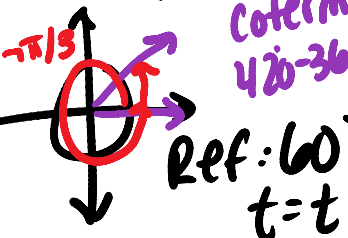
a.) $-\frac{3\pi}{5}$ $-\frac{3\pi}{5} + \frac{10\pi}{5} = \frac{7\pi}{5}$



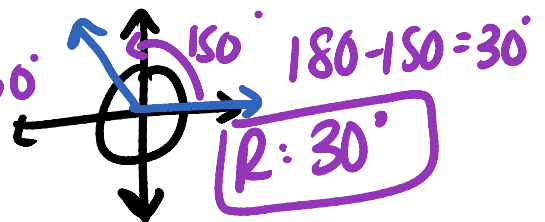
$\frac{7\pi}{5} \cdot \frac{180}{\pi} = 252^\circ$

Ref: 12°

b.) $\frac{7\pi}{3}$ $\frac{7\pi}{3} \cdot \frac{180}{\pi} = 420^\circ$

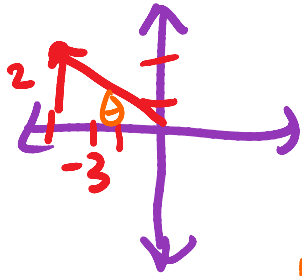


c.) 150°



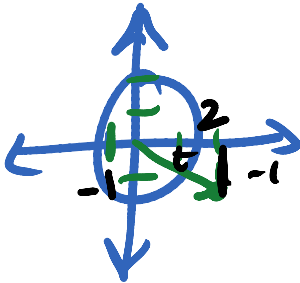
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6. Find the sine, cosine, and tangent of the angle, when the terminal side passes through $(-3, 2)$.



$$\begin{aligned} \sin \theta &= \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13} & 2^2 + (-3)^2 &= r^2 \\ \cos \theta &= \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13} & 4 + 9 &= r^2 \\ \tan \theta &= \frac{2}{-3} = -\frac{2}{3} & 13 &= r^2 \\ & & r &= \sqrt{13} \end{aligned}$$

7. The terminal side of an angle of t radians lies in quadrant II on a line through the origin parallel to $2y + x = 6$. Find $\cos t$.



$$\begin{aligned} 2y &= 6 - x \\ y &= 3 - \frac{1}{2}x \\ m &= -\frac{1}{2} \end{aligned} \quad \begin{aligned} 2^2 + (-1)^2 &= r^2 \\ 4 + 1 &= r^2 \\ 5 &= r^2 \\ r &= \sqrt{5} \end{aligned}$$

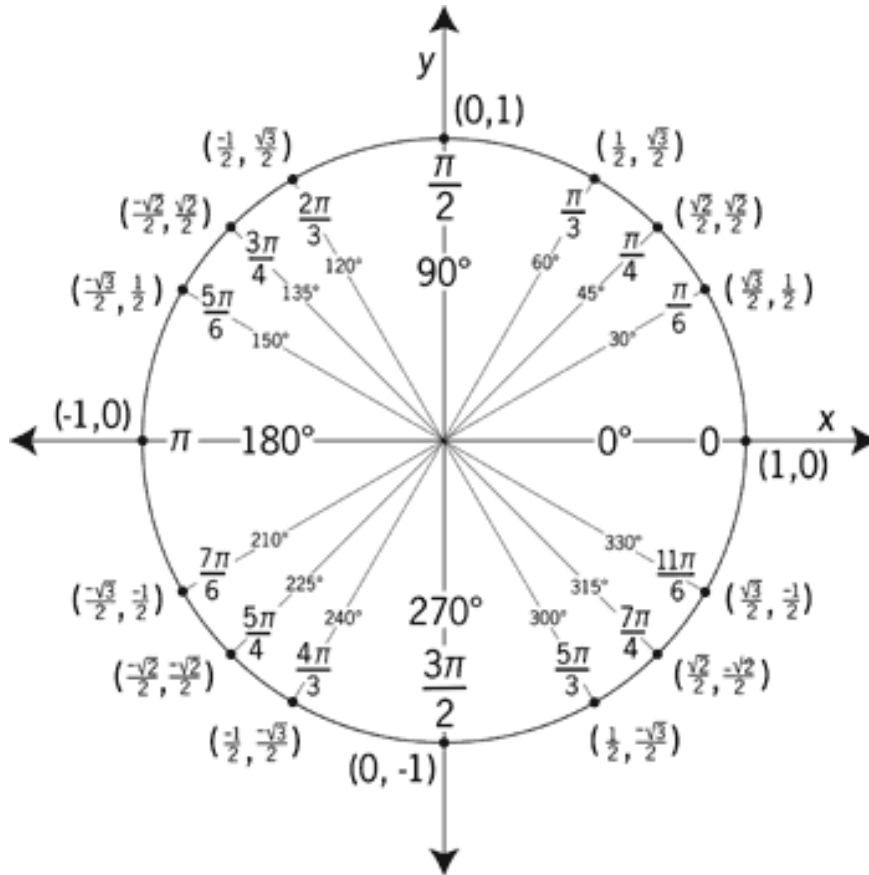
$$\cos t = \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$$

8. Express a single real number:

$$\begin{aligned} \left(\sin \frac{\pi}{6} + 1\right)^2 & \quad \sin \frac{\pi}{6} = \frac{1}{2} \\ \left(\frac{1}{2} + 1\right)^2 &= \boxed{\frac{9}{4}} \end{aligned}$$

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9. We now have all the tools we need to complete the unit circle. The unit circle either has a radian measure at each point or a degree measure, your job is to convert the angle measure, and then algebraically find the coordinate point that corresponds with that angle measure.



Convert between degrees & radians:

$$\frac{\pi}{180} \quad \text{or} \quad \frac{180}{\pi}$$

To find coordinate points:

$(\cos\theta, \sin\theta) \rightarrow (x, y)$

