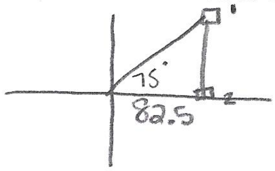
Pre-Calculus

9-2 Addition and Subtraction Identities

Name Key!!

1. While on the Ferris wheel at Six Flags, you are in a car that has an angle of elevation with the center of the wheel of 75°. Your friend is in a car on the positive x-axis of the wheel. The radius of the Ferris wheel is 82.5 feet. Find the vertical distance to your friend if the wheel is stopped. Find an exact answer if possible. If not, explain why you cannot find an exact answer.



$\tan 75 = \frac{x}{82.5}$

$82.5 \tan 75 = x$

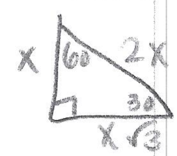
$x = 307.89$ feet

→ Can't find exact values because not on the unit circle.

2. Show how you can rewrite 75° so it relates two angles that are on the unit circle.

$75° = 45° + 30°$

$\tan 75° = \tan(45 + 30°)$



3. Show how you can rewrite $\frac{17\pi}{12}$ as the sum of two angles on the unit circle.

$\frac{17\pi}{12} = \frac{(3)(3\pi)}{4} + \frac{(4)(2\pi)}{3} \rightarrow \frac{17\pi}{12} = \frac{3\pi}{4} + \frac{2\pi}{3}$

4. Show how you can rewrite $\frac{7\pi}{12}$ as both the sum and difference of two angles on the unit circle.

Sum: $\frac{7\pi}{12} \rightarrow \frac{\pi}{3} + \frac{\pi}{4}$

Difference: $\frac{7\pi}{12} \rightarrow (4)\frac{4\pi}{3} - \frac{3\pi}{4}$

Calc in radian mode. →

5. Could sin(x + 5) = sin x + sin 5 possibly be an identity? How do you know?

– Can't be an identity because the graphs are not the same.

6. Could cos(x – 3) = cos x – cos 3 possibly be an identity? How do you know?

– no. because the graphs are not the same.

7. Could cos(x + 4) = cox x(cos 4) – sin x(sin 4) possibly be an identity? How do you know?

yes this can possibly be an identity judging from the graphs.

8. Write a cosine function whose graph looks like the graph of f(x) = sin x

$g(x) = \cos(x - \pi/2)$

→ shifted over $\pi/2$.

Addition and Subtraction Identities:

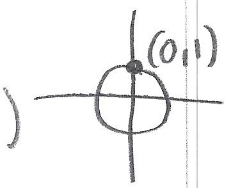
Sin(x + y) = sin x cos y + cos x sin y

Sin (x – y) = sin x cos y – cos x sin y

Cos (x + y) = cos x cos y – sin x sin y

Cos (x – y) = cos x cos y + sin x sin y

Use the appropriate identity to simplify the following



9. Cos $(x - \frac{\pi}{2})$ $= \cos x \cos(\pi/2) + \sin x \sin(\pi/2)$

$= \cancel{\cos x (0)} + \sin x (1)$

$= \boxed{\sin x}$

- How does your answer from #8 relate to your answer for #9?

→ the cosine function that I found in 8 showed that $\cos(x - \pi/2) = \sin x$. I found this result algebraically in #9.

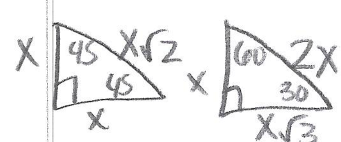
10. Cos$(x + \frac{\pi}{2})$ $= \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$

$= \cos x (0) - \sin x (1)$

$= -\sin x$

Use the appropriate identity to evaluate. **You must get an exact answer.**

11. Sin 75°



$\sin(45+30) = \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

12. Cos $\frac{17\pi}{12}$ $= \frac{17\pi}{12} = \frac{15\pi}{12} + \frac{2\pi}{12}$

$= \frac{5\pi}{4} + \frac{\pi}{6}$

$\cos\left(\frac{5\pi}{4} + \frac{\pi}{6}\right) = \cos\left(\frac{5\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\frac{5\pi}{4}\sin\frac{\pi}{6}$

$= -\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) - \left(+\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \sqrt{2} - \sqrt{6}$

13. $\sin\frac{17\pi}{12} = \frac{15\pi}{12} + \frac{2\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$

$\sin\left(\frac{5\pi}{4} + \pi/6\right) = \sin\frac{5\pi}{4}\cos\frac{\pi}{6} + \cos\frac{5\pi}{4}\sin\frac{\pi}{6}$

$= -\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + -\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{6}-\sqrt{2}}{4}$

14. $\tan\frac{7\pi}{12} = \frac{15\pi}{12} + \frac{2\pi}{12} = \frac{5\pi}{4} + \frac{\pi}{6}$ $\quad -2-\sqrt{3}$

$\tan\left(\frac{5\pi}{4} + \frac{\pi}{6}\right) = \frac{\sin\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)}{\cos\left(\frac{5\pi}{4} + \frac{\pi}{6}\right)} = \frac{\frac{-\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$= \frac{-\sqrt{6}-\sqrt{2}}{\cancel{4}} \cdot \frac{\cancel{4}}{\sqrt{2}-\sqrt{6}} = \frac{-\sqrt{6}-\sqrt{2}}{\sqrt{2}-\sqrt{6}} = \frac{\cancel{\sqrt{2}}(-\sqrt{3}-1)}{\cancel{\sqrt{2}}(1-\sqrt{3})}$

$= \frac{(-\sqrt{3}-1)}{(1-\sqrt{3})} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{-\sqrt{3}-1-3-\sqrt{3}}{1-3} = \frac{-2\sqrt{3}-4}{-2}$

$= \frac{-2(\sqrt{3}+2)}{-2} = \boxed{2+\sqrt{3}}$

15. Simplify: sin(x + y) – sin(x – y)

$= (\sin x\cos y + \cos x\sin y) - (\sin x\cos y - \cos x\sin y)$

$= 2\cos x\sin y$ ✓