9.4 Homework pg. 608 # 1-17odd, 23-24, 29, 34

1. $\sin^2 x + 3\cos^2 x = 0 \qquad [0, 2\pi]$

$(1-\cos^2 x) + 3\cos^2 x = 0$

$1 + 2\cos^2 x = 0$

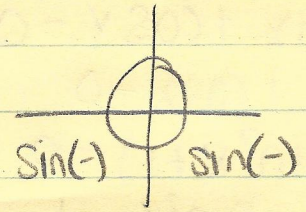
$\frac{2\cos^2 x}{2} = \frac{-1}{2} \Rightarrow \sqrt{\cos^2 x} = \sqrt{\frac{-1}{2}}$ no solution

3 $\cos 2x - \sin x = 1 \qquad \cos 2x = 1 - 2\sin^2 x$

$1 - 2\sin^2 x - \sin x = 1$

$-2\sin^2 x - \sin x = 0$

$\sin x(-2\sin x - 1) = 0$



$\sin x = 0 \qquad -2\sin x - 1 = 0$

$x = 0, \pi, 2\pi \qquad \frac{-2\sin x}{-2} = \frac{1}{-2} \qquad \sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

5. $4\sin^2\left(\frac{x}{2}\right) + \cos^2 x = 2$

$4\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 + \cos^2 x = 2$

$4\left(\frac{1-\cos x}{2}\right) + \cos^2 x = 2$

$2(1-\cos x) + \cos^2 x = 2$

$2 - 2\cos x + \cos^2 x = 2$

$-2\cos x + \cos^2 x = 0 \Rightarrow \cos^2 x - 2\cos x = 0$

$\cos x(\cos x - 2) = 0$

$\cos x = 0 \qquad \cos x = 2$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ no solution

~~7.~~ $\sin x \sin \frac{1}{2}x = 1-\cos x$

$\left(\pm \sin x \sqrt{\frac{1-\cos x}{2}}\right)^2 = (1-\cos x)^2$

$\frac{\sin^2 x \left(\frac{1-\cos x}{2}\right)}{1-\cos x} = \frac{(1-\cos x)^2}{1-\cos x}$

9. $\sin 2x \sin x + \cos x = 0$

$2\sin^2 x \cos x + \cos x = 0$

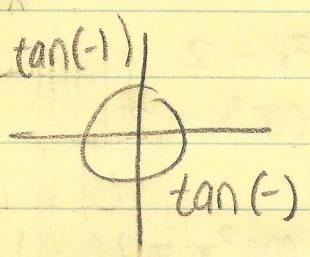
$\cos x(2\sin^2 x + 1) = 0$

$\cos x = 0 \qquad \sin^2 x = \frac{-1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ no solution

11. $\sin 2x + \cos 2x = 0$

$\frac{\sin 2x}{\cos 2x} = \frac{-\cos 2x}{\cos 2x}$



$\tan 2x = -1 \qquad U = 2x$

$\tan U = -1$

$U = \frac{3\pi}{4}, \frac{7\pi}{4}$

$2x = \frac{3\pi}{4} \quad \frac{7\pi}{4} + 2\pi k$

$x = \frac{3\pi}{8} + \pi k, \frac{7\pi}{8} + \pi k, \frac{11\pi}{8}, \frac{15\pi}{8}$ ← within 2π

13. $\sin 4x = \cos 2x$

$2\sin 2x \cos 2x = \cos 2x$

$2\sin 2x \cos 2x - \cos 2x = 0$

$\cos 2x(2\sin 2x - 1) = 0$

$\cos 2x = 0 \qquad 2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$

$\cos 2x = 0 \quad U = 2x$

$\cos U = 0$

$U = \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{2x}{2} = \frac{\frac{\pi}{2}, \frac{3\pi}{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4} + \pi k$

$\Rightarrow \frac{5\pi}{4}, \frac{7\pi}{4} \to 2\pi$

$\sin 2x = \frac{1}{2} \quad U = 2x$

$\sin U = \frac{1}{2}$

$U = \frac{\pi}{6}, \frac{5\pi}{6}$

$2x = \frac{\pi}{12}, \frac{5\pi}{12} + \pi k$

$\frac{17\pi}{12}, \frac{3\pi}{12} \to 2\pi$

15. $2\cos^2 x - 2\cos 2x = 1$

$2\cos^2 x - 2(1 - 2\cos^2 x) = 1$

$2\cos^2 x - 2 + 4\cos^2 x = 1$

$6\cos^2 x = 3$

$\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$

$\cos x = \pm\frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

17. $\sin\left(x + \frac{\pi}{2}\right) + \cos x = 1$ (1)

$\sin x \cdot \cos\left(\frac{\pi}{2}\right) + \cos x \sin\frac{\pi}{2} + \cos x = 1$

$\cos x + \cos x = 1 \Rightarrow 2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

23. $\sin x \cos x + \frac{1}{2} = 0$

$2(\sin x \cos x) = \left(-\frac{1}{2}\right)2$

$2\sin x \cos x = -1 \quad U = 2x$

$\sin 2x = -1$

$\sin U = -1$

$U = \frac{3\pi}{2} \rightarrow 2x = \frac{3\pi}{2} \cdot \frac{1}{2} + \frac{2\pi k}{2}$

$x = \frac{3\pi}{4} + \pi k$; $\frac{7\pi}{4}$ within 2π

24. $\sin^2\left(\frac{x}{2}\right) + \cos x = 0$

$\left(\sqrt{\frac{1-\cos x}{2}}\right)^2 + \cos x$

$\frac{1-\cos x}{2} + \cos x(2) = 0$

$1 - \cos x + 2\cos x = 0$

$1 + \cos x = 0$

$\cos x = -1$

$x = \pi$

29. $\sin^2 x + \cos 2x = 1$

$(1-\cos^2 x) + \cos^2 x - \sin^2 x = 1$

$1 - \sin^2 x = 1$

$-\sin^2 x = 0$

$\sin x = 0 \quad x = 0, \pi, 2\pi \Rightarrow$ between $[-\pi, \pi]$ is $-\pi, 0, \pi$

34. $\cos 3x + \cos x = 0$

$\cos(2x + x) + \cos(x)$

$\cos 2x \cos x - \sin 2x \sin x + \cos x$