Ch.9 Review Solutions

Wednesday, April 16, 2014
3:30 PM

$-x)$ $\quad \cos x = \sin\left(\frac{\pi}{2} - x\right)$

$-x)$ $\quad \cot x = \tan\left(\frac{\pi}{2} - x\right)$

$-x)$ $\quad \csc x = \sec\left(\frac{\pi}{2} - x\right)$

$x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\sqrt{\frac{1 - \cos x}{2}}$

$\sqrt{\frac{1 + \cos x}{2}}$

$\frac{- \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$

m expression.

$\cos^2 t$

3. $\frac{\tan^2 x - \sin^2 x}{\sec^2 x}$

1

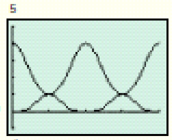
lically whether the equation could not pos-
of showing that it is.

6. $1 + 2\cos^2 t + \cos^4 t = \sin^4 t$

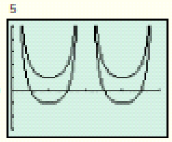
8. $\frac{\sin^2 t}{\cos^2 t} + 1 = \frac{1}{\cos^2 t}$

10. $\tan x + \cot x = \sec x \csc x$

1. $\frac{1}{3} + \cot t$
2. $\tan t$
3. $\sin^4 x$
4. 2
5. $\sin^4 t - \cos^4 t$
   $= (\sin^2 t - \cos^2 t)(\sin^2 t + \cos^2 t)$
   $= [\sin^2 t - (1 - \sin^2 t)](1)$
   $= 2\sin^2 t - 1$
6. not an identity



7. $\frac{\sin t}{1 - \cos t} = \frac{\sin t}{1 - \cos t} \cdot \frac{(1 + \cos t)}{(1 + \cos t)}$
   $= \frac{\sin t(1 + \cos t)}{1 - \cos^2 t}$
   $= \frac{\sin t(1 + \cos t)}{\sin^2 t}$
   $= \frac{1 + \cos t}{\sin t}$
8. $\frac{\sin^2 t}{\cos^2 t} + 1 = \frac{\sin^2 t + \cos^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$
9. not an identity



10. $\tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$
    $= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$
    $= \frac{1}{\cos x \sin x}$
    $= \frac{1}{\cos x} \cdot \frac{1}{\sin x}$
    $= \sec x \csc x$
11. $(\sin x + \cos x)^2 - \sin 2x$
    $= \sin^2 x + 2\sin x \cos x +$
    $\cos^2 x - 2\sin x \cos x$
    $= \sin^2 x + \cos^2 x = 1$
12. $\frac{\sec x + 1}{\tan x} = \frac{(\sec x + 1)(\sec x - 1)}{\tan x(\sec x - 1)}$
    $= \frac{\sec^2 x - 1}{\tan x(\sec x - 1)}$
    $= \frac{\tan^2 x}{\tan x(\sec x - 1)}$
    $= \frac{\tan x}{\sec x - 1}$

13. $\frac{\cos^4 x - \sin^4 x}{1 - \tan^4 x}$
    $= \frac{(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)}{(1 - \tan^2 x)(1 + \tan^2 x)}$
    $= \frac{\cos^2 x - \sin^2 x}{(1 - \tan^2 x)\sec^2 x}$
    $= \frac{\cos^2 x - \sin^2 x}{\left(1 - \frac{\sin^2 x}{\cos^2 x}\right)\frac{1}{\cos^2 x}}$
    $= \frac{(\cos^2 x - \sin^2 x)\cos^4 x}{\cos^2 x - \sin^2 x} = \cos^4 x$

14. $\frac{1 + \tan^2 x}{\tan^2 x} = \frac{1}{\tan^2 x} + \frac{\tan^2 x}{\tan^2 x}$
    $= \frac{1}{\tan^2 x} + 1$
    $= \cot^2 x + 1$
    $= \csc^2 x$
15. $\sec x - \cos x = \frac{1}{\cos x} - \cos x$
    $= \frac{1 - \cos^2 x}{\cos x}$
    $= \frac{\sin^2 x}{\cos x}$
    $= \sin x \frac{\sin x}{\cos x}$
    $= \sin x \tan x$
16. $\tan^2 x - \sec^2 x$
    $= (\sec^2 x - 1) - \sec^2 x = -1$
    Also, $\cot^2 x - \csc^2 x$
    $= (\csc^2 x - 1) - \csc^2 x = -1.$
    Therefore,
    $\tan^2 x - \sec^2 x = \cot^2 x - \csc^2 x.$
17. a
18. b
19. $\cos(x + y)\cos(x - y)$
    $= [\cos x \cos y - \sin x \sin y] \cdot [\cos x \cos y + \sin x \sin y]$
    $= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$
    $= \cos^2 x(1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y$
    $= \cos^2 x - \cos^2 x \sin^2 y - \sin^2 y + \cos^2 x \sin^2 y$
    $= \cos^2 x - \sin^2 y$
20. $\frac{\cos(x - y)}{\cos x \cos y} = \frac{\cos x \cos y + \sin x \sin y}{\cos x \cos y}$
    $= \frac{\cos x \cos y}{\cos x \cos y} + \frac{\sin x \sin y}{\cos x \cos y}$
    $= 1 + \frac{\sin x \sin y}{\cos x \cos y}$
    $= 1 + \tan x \tan y$
21. a. $-\frac{3}{5}$
    b. $\frac{117}{44}$
    c. $\frac{44}{125}$
22. $-\frac{56}{65}$
23. $-\frac{120}{169}$
24. $\frac{3\sqrt{5} + 1}{8}$
25. $\frac{\sqrt{42} + 2\sqrt{2}}{10}$
26. $\frac{\sqrt{2} + \sqrt{6}}{4}$
27. $-\frac{1}{\cos x}$

In Exercises 12–16, prove the given identity.

12. $\frac{\sec x + 1}{\tan x} = \frac{\tan x}{\sec x - 1}$
13. $\frac{\cos^4 x - \sin^4 x}{1 - \tan^4 x} = \cos^4 x$
14. $\frac{1 + \tan^2 x}{\tan^2 x} = \csc^2 x$
15. $\sec x - \cos x = \sin x \tan x$
16. $\tan^2 x - \sec^2 x = \cot^2 x - \csc^2 x$
17. $\sqrt{\frac{1 - \cos^2 x}{1 - \sin^2 x}} = ?$
    a. $|\tan x|$
    b. $|\cot x|$
    c. $\sqrt{\frac{1 - \sin^2 x}{1 - \cos^2 x}}$
    d. $\sec x$
    e. undefined
18. $\frac{1}{(\csc x)(\sec^2 x)} = ?$
    a. $\frac{1}{(\sin x)(\cos^2 x)}$
    b. $\sin x - \sin^3 x$
    c. $\frac{1}{(\sin x)(1 + \tan^2 x)}$
    d. $\sin x - \frac{1}{1 + \tan^2 x}$
    e. $1 + \tan^2 x$

Section 9.2

In Exercises 19–20, prove the given identity.

19. $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$
20. $\frac{\cos(x - y)}{\cos x \cos y} = 1 + \tan x \tan y$
21. Evaluate the following in exact form, where the angles α and β satisfy t
    conditions:
    $\sin\alpha = \frac{4}{5}$ for $\frac{\pi}{2} < \alpha < \pi$ $\qquad$ $\tan\beta = \frac{7}{24}$ for $\pi < \beta < \frac{3\pi}{2}$.
    a. $\sin(\beta + \alpha)$ $\quad$ b. $\tan(\beta - \alpha)$ $\quad$ c. $\cos(\alpha - \beta)$
22. If $\tan x = \frac{4}{3}$ and $\pi < x < \frac{3\pi}{2}$, and $\cot y = -\frac{5}{12}$ with $\frac{3\pi}{2} < y < 2\pi$, fi
    $\sin(x - y)$.
23. If $\sin x = -\frac{12}{13}$ with $\pi < x < \frac{3\pi}{2}$, and $\sec y = \frac{13}{12}$ with $\frac{3\pi}{2} < y < 2\pi$,
    $\cos(x + y)$.
24. If $\sin x = \frac{1}{4}$ and $0 < x < \frac{\pi}{2}$, then $\sin\left(\frac{\pi}{3} + x\right) = ?$
25. If $\sin x = -\frac{2}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cos\left(\frac{\pi}{4} + x\right) = ?$
26. Find the exact value of $\sin\frac{5\pi}{12}$.

28. $\frac{\pi}{4}$
29. ≈ 1.23 radians
30. a. $\frac{\sqrt{226}}{226}$
    b. $\frac{11{,}753}{15{,}625}$
    c. $\frac{-10{,}296}{11{,}753}$
    d. $\frac{-26{,}208}{14{,}125}$
31. $\frac{1 - \cos 2x}{\tan x} = \frac{1 - (1 - 2\sin^2 x)}{\tan x}$
    $= \frac{2\sin^2 x}{\tan x}$
    $= \frac{2\sin^2 x}{\frac{\sin x}{\cos x}}$
    $= \frac{2\sin^2 x \cos x}{\sin x}$
    $= 2\sin x \cos x$
    $= \sin 2x$

27. Express $\sec(x - \pi)$ in terms of $\sin x$ and $\cos x$.

Section 9.2.A

28. Find the angle of inclination of the straight line through the points (2, 6) and (−2, 2).

29. Find one of the angles between the line L through the points (−3, 2) and (5, 1) and the line M, which has slope 2.

Section 9.3

30. Evaluate the following in exact form, where the angles α and β satisfy the conditions:

$$\sin\alpha = \frac{44}{125} \text{ for } \frac{\pi}{2} < \alpha < \pi \qquad \tan\beta = -\frac{15}{112} \text{ for } \frac{3\pi}{2} < \beta < 2\pi$$

a. $\sin\frac{\beta}{2}$ b. $\cos 2\alpha$

c. $\tan 2\alpha$ d. $\cos(\alpha - \beta) + \cos(\alpha + \beta)$

In Exercises 31–34, prove the given identity.

31. $\dfrac{1 - \cos 2x}{\tan x} = \sin 2x$

32. $\dfrac{\tan x - \sin x}{2\tan x} = \sin^2\dfrac{x}{2}$

33. $2\cos x - 2\cos^3 x = \sin x \sin 2x$

34. $\sin 2x = \dfrac{1}{\tan x + \cot 2x}$

35. If $\tan x = \frac{5}{12}$ and $\sin x > 0$, find $\sin 2x$.

36. If $\cos x = \frac{15}{17}$ and $0 < x < \frac{\pi}{2}$, find $\sin\frac{x}{2}$.

37. If $\sin x = 0$, is it true that $\sin 2x = 0$? Justify your answer.

38. If $\cos x = 0$, is it true that $\cos 2x = 0$? Justify your answer.

39. Show $\sqrt{2 + \sqrt{3}} = \dfrac{\sqrt{2} + \sqrt{6}}{2}$ by computing $\cos\frac{\pi}{12}$ in two ways, using the half-angle identity and the subtraction identity for cosine.

40. True or false: $2\sin x = \sin 2x$. Justify your answer.

41. If $\sin x = 0.6$ and $0 < x < \frac{\pi}{2}$, find $\sin 2x$.

42. If $\sin x = 0.6$ and $0 < x < \frac{\pi}{2}$, find $\sin\frac{x}{2}$.

Section 9.4

Solve the equation. Find exact solutions when possible and approximate ones otherwise.

43. $5\tan x = 2\sin 2x$

44. $\cos 2x = \cos x$

45. $2\cos x + \sin x = 0$

46. $\sin 2x + \cos x = 0$

40. False. $\sin\left(2 \cdot \frac{\pi}{2}\right) = 0$, but $2\sin\frac{\pi}{2} = 2$

41. 0.96

42. $\dfrac{1}{\sqrt{10}}$

43. $x = k\pi$

44. $x = \frac{2\pi}{3} + 2k\pi, \frac{4\pi}{3} + 2k\pi, 2k\pi$

45. $x \approx 2.0344 + k\pi$

46. $x = \frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi, \frac{7\pi}{6} + 2k\pi, \frac{11\pi}{6} + 2k\pi$

32.
$$\frac{\tan x - \sin x}{2\tan x} = \frac{\tan x}{2\tan x} - \frac{\sin x}{2\tan x}$$
$$= \frac{1}{2} - \frac{\sin x}{\frac{2\sin x}{\cos x}}$$
$$= \frac{1}{2} - \sin x\left(\frac{\cos x}{2\sin x}\right)$$
$$= \frac{1}{2} - \frac{\cos x}{2}$$
$$= \frac{1 - \cos x}{2}$$
$$= \left(\pm\sqrt{\frac{1 - \cos x}{2}}\right)^2$$
$$= \left(\sin\left(\frac{x}{2}\right)\right)^2$$
$$= \sin^2\left(\frac{x}{2}\right)$$

33.
$$2\cos x - 2\cos^3 x$$
$$= 2\cos x(1 - \cos^2 x)$$
$$= 2\cos x(\sin^2 x)$$
$$= \sin x(2\sin x\cos x)$$
$$= \sin x \sin 2x$$

34.
$$\frac{1}{\tan x + \cot 2x}$$
$$= \frac{1}{\tan x + \frac{\cos 2x}{\sin 2x}}$$
$$= \frac{\sin 2x}{\sin 2x \tan x + \cos 2x}$$
$$= \frac{\sin 2x}{\left(\frac{\sin x}{\cos x}\right)2\sin x\cos x + \cos 2x}$$
$$= \frac{\sin 2x}{2\sin^2 x + (1 - 2\sin^2 x)}$$
$$= \frac{\sin 2x}{1} = \sin 2x$$

35. $\dfrac{120}{169}$

36. $\dfrac{1}{\sqrt{17}}$

37. Yes. $\sin 2x = 2\sin x\cos x = 2(0)\cos x = 0$

38. No. $\cos 2x = 2\cos^2 x - 1 = 2(0^2) - 1 = -1$

39.
$$\cos\left(\frac{\pi}{12}\right) = \cos\left[\frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$
$$= \sqrt{\frac{1 + \cos\frac{\pi}{6}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}}$$
$$= \sqrt{\frac{2 + \sqrt{3}}{4}} = \frac{\sqrt{2 + \sqrt{3}}}{2};$$
$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$
$$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$
$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2}\left(\frac{1}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$$

So, $\dfrac{\sqrt{2 + \sqrt{3}}}{2} = \dfrac{\sqrt{6} + \sqrt{2}}{4}$ or $\sqrt{2 + \sqrt{3}} = \dfrac{\sqrt{2} + \sqrt{6}}{2}$