Solutions:Trig Identities and Proof

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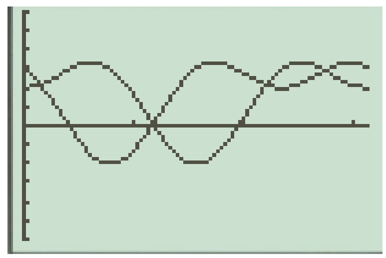
1. Is either of the following equations and identity?

a.) $3sin^2x + 2cosx = 3cos^2x - 2sinx$

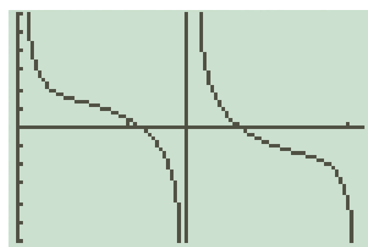
→ NO because the graphs of the equations do not appear to be the same.



b.) $\frac{1+cosx-cos^2x}{sinx} = sinx + cotx$

The graphs appear to be the same.

→ Therefore, the equation <u>may</u> be an identity.



2. Verify algebraically that: $\frac{1+cosx-cos^2x}{sinx} = sinx + cotx$

$sin^2x + cos^2x = 1$

$$\frac{(1-cos^2x)+cosx}{sinx} =$$

$\sin^2 x + \cos^2 x = 1 \qquad \frac{(1-\cos^2 x)+\cos x}{\sin x} =$

$$\frac{\sin^2 x + \cos x}{\sin x} =$$

$$\frac{\sin^{\cancel{2}} x}{\cancel{\sin x}} + \frac{\cos x}{\sin x} =$$

$$\sin x + \cot x = \sin x + \cot x$$

3. Simplify $(secx + tanx)(1 - sinx)$

$$\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)(1-\sin x)$$

$$\left(\frac{1+\sin x}{\cos x}\right)(1-\sin x)$$

$$\frac{(1+\sin x)(1-\sin x)}{\cos x}$$

$$\frac{1 \cancel{+\sin x} \cancel{-\sin x} - \sin^2 x}{\cos x} = \frac{1-\sin^2 x}{\cos x}$$

$$\frac{\cos^{\cancel{2}} x}{\cancel{\cos x}} = \boxed{\cos x}$$

4. Prove that $\frac{cosx}{1-sinx} = \frac{1+sinx}{cosx}$

① $$\frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} =$$

$1-\sin x \quad 1+\sin x$

$$\frac{\cos x(1+\sin x)}{(1-\sin x)(1+\sin x)} =$$

Pythagorean Identity

$\cos^2 x + \sin^2 x = 1$

$$\frac{\cos x(1+\sin x)}{1-\sin^2 x} =$$

$$\frac{\cancel{\cos x}(1+\sin x)}{\cos^2 x}$$

$$\frac{(1+\sin x)}{\cos x} = \frac{(1+\sin x)}{\cos x}$$

2. $$\frac{\cos x}{1-\sin x} \cdot \frac{\cos x}{\cos x} = \frac{\cos^2 x}{\cos x(1-\sin x)}$$

Pythagorean Identity

$\sin^2 x + \cos^2 x = 1$

$$= \frac{1-\sin^2 x}{\cos x(1-\sin x)}$$

$$= \frac{\cancel{(1-\sin x)}(1+\sin x)}{\cos x\cancel{(1-\sin x)}} = \frac{1+\sin x}{\cos x}$$

5. Prove that $secx + tanx = \frac{cosx}{1-sinx}$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} =$$

From Example above,

$$\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$$

from above

$$\boxed{\frac{1+\sin x}{\cos x}} = \frac{\cos x}{1-\sin x}$$

Thus, $\sec x + \tan x = \frac{\cos x}{1-\sin x}$

Thus, $\sec x + \tan x = \frac{\cos x}{1 - \sin x}$

6. Prove that $\csc x(cscx - sinx) = cot^2 x$

Pythagorean Identity:

$1 + \cot^2 x = \csc^2 x$

$\underline{\cot^2 x} = \csc^2 x - 1$

$\csc^2 x - \csc x \sin x =$

$\csc^2 x - \frac{1}{\cancel{\sin x}} \cdot \cancel{\sin x} =$

$\csc^2 x - 1 =$

$\cot^2 x = \cot^2 x$

Not cross multiplying. Then assume identity is true.

7. Prove that $\frac{cscx}{cotx} \cancel{=} \frac{cotx}{cscx - sinx}$

use the identity above.

$\csc(\csc x - \sin x) = \cot^2 x$ ← from #6.

$\frac{\csc x(\cancel{\csc x - \sin x})}{\cot x(\cancel{\csc x - \sin x})} = \frac{\cot^{\cancel{2}} x}{\cancel{\cot x}(\csc x - \sin x)}$ ← don't assume, have to prove

$\frac{\csc x}{\cot x} = \frac{\cot x}{\csc x - \sin x}$

★ Trying to prove an identity w/o fractions.

Not cross multiplying

8. Prove that $\frac{cosx-1}{cosx+1} = \frac{1-secx}{1+secx}$

★ To prove an identity involving fractions, you need to prove a different identity not involving fractions

① $(\cos x - 1)(1 + \sec x) = (\cos x + 1)(1 - \sec x)$

$\cos x + \cos x \sec x - \sec x - 1 =$

$$\cos x + \cos x \sec x - \sec x - 1 =$$

$$\cos x + \cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} - \sec x - 1 =$$

$$\cos x + \cancel{1} - \sec x - \cancel{1} =$$

$$\cos x - \sec x =$$

2) $(\cos x + 1)(1 - \sec x)$

$$\cos x + 1 - \cos x \sec x - \sec x =$$

$$\cos x + 1 - \cancel{\cos x} \cdot \frac{1}{\cancel{\cos x}} - \sec x =$$

$$\cos x + \cancel{1} - \cancel{1} - \sec x$$

$$= \cos x - \sec x$$

Thus, $\cos x - \sec x = \cos x - \sec x$